Code No: 131AA **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February - 2024
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Find the orthogonal trajectories of the family of semi cubical parabolas $ay^2 = x^3$. [2]

b) Solve $D^2x + 6Dx + 9x = 0$, where D = $\frac{\partial}{\partial t}$. [3]

c) Define orthogonal matrix and give an example. [2]

d) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. [3]

e) State Cayley-Hamilton theorem. [2]

f) Prove that the eigen values of an idempotent matrix is either zero or unity. [3]

g) If $z = f(x + ct) + \phi(x - ct)$ then show that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$. [2]

h) If $u = F(x - y, y - z, z - x)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. [3]

i) Find the Partial differential equation by eliminating arbitrary function f from $z = f(x^2 + y^2)$. [2]

j) Solve the equation $xp + yq = 3z$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [3]

PART - B

(50 Marks)

2.a) Solve $(D^2 - 1)y = xsin3x + cosx$.

b) Solve by the method of variation of parameters $y'' - 2y' + y = e^x logx$. [5+5]

OR

3.a) Solve $(D^2 - 2D + 1)y = xe^x sinx$.

b) A body originally at 80^0C cools down to 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original? [5+5]

4.a) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) many solutions.

b) Use Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. [5+5]

OR

5.a) Solve the system of equations by Gauss Elimination Method
$x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40.$

b) Solve the following system of equations using LU Decomposition method [5+5]
$x + 2y + 3z = 14, 2x + 3y + 4z = 20, 3x + 4y + z = 14.$

6.a) Find the eigen values and the eigen vectors for the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

b) Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$ to canonical form using orthogonal transformation. [5+5]

OR

7.a) Using Cayley-Hamilton theorem, evaluate A^{-1}, A^{-2} and A^{-3} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.

b) Diagonalize the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. [5+5]

8.a) If $u = sin^{-1}(\frac{x^2y^2}{x+y})$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\, tanu$.

b) If $u = x\sqrt{(1 - y^2)} + y\sqrt{(1 - x^2)}$, $v = \sin^{-1} x + \sin^{-1} y$, show that u, v are functionally related and find the relation. [5+5]

OR

9.a) Find the Taylor series expansion of $f(x, y) = e^x cosy$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{4}\right)$.

b) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. [5+5]

10.a) Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = (ly - mx)$.

b) Solve $(p^2 + q^2)y = qz$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [5+5]

OR

11.a) Solve $2xz - px^2 - 2qxy + pq = 0$.

b) Find the complete integrals of the equations $p + q = pq$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [5+5]

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